POLITECHNIKA GDAŃSKA
CENTRUM MATEMATYKI

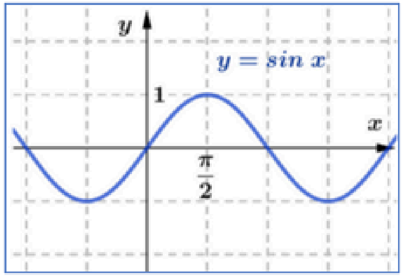
Zestaw podstawowych wzorów

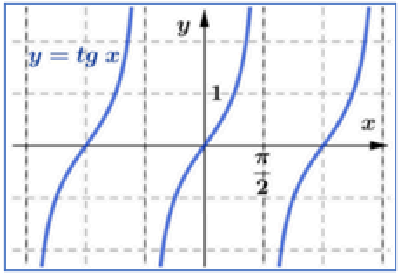
Wzory skróconego mnożenia

Kwadrat sumy: $(a+b)^2 = a^2 + 2ab + b^2$ Kwadrat różnicy: $(a-b)^2 = a^2 - 2ab + b^2$	Suma kwadratów: $a^2 + b^2 = (a+bi)(a-bi)$ Różnica kwadratów: $a^2 - b^2 = (a+b)(a-b)$
Sześcian sumy: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ Sześcian różnicy: $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$	Suma sześcianów: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ Różnica sześcianów: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Logarytmy

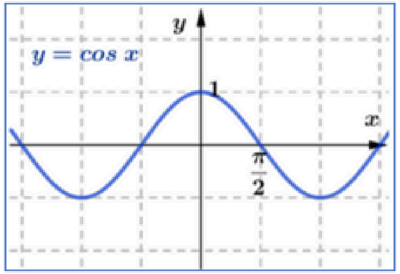
$\log_a b = c \ \Leftrightarrow b = a^c, \qquad a\,\epsilon(0,1) \cup (1,\infty), b\epsilon(0,\infty)$	$\log_a b^n = n \cdot \log_a b, \quad \log_a b = \frac{\log_c b}{\log_c a}, \quad a^{\log_a b} = b$
Suma logarytmów: $\log_a b_1 + \log_a b_2 = \log_a(b_1 \cdot b_2)$	Różnica logarytmów: $\log_a b_1 - \log_a b_2 = \log_a \frac{b_1}{b_2}$
$a\epsilon(0,1) \Longrightarrow (\log_a b_1 < \log_a b_2 \ \Leftrightarrow b_1 > b_2)$	$a\epsilon(1,\infty) \Longrightarrow (\log_a b_1 < \log_a b_2 \ \Leftrightarrow b_1 < b_2)$

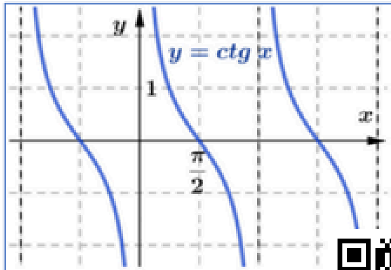




Trygonometria

$\sin^2(x) = \frac{1-\cos(2x)}{2}, \quad \cos^2(x) = \frac{1+\cos(2x)}{2}$
$\sin\alpha + \sin\beta = 2 \cdot \sin\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2}, \quad \alpha,\beta\epsilon\mathbb{R}$
$\sin\alpha - \sin\beta = 2 \cdot \cos\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2}, \quad \alpha,\beta\epsilon\mathbb{R}$
$\cos\alpha + \cos\beta = 2 \cdot \cos\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2}, \quad \alpha,\beta\epsilon\mathbb{R}$
$\cos\alpha - \cos\beta = -2 \cdot \sin\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2}, \quad \alpha,\beta\epsilon\mathbb{R}$





Wzory na pochodne

$f(x)$	$f'(x)$
Niech: $a \epsilon \mathbb{R}$, $b \epsilon (0,1) \cup (1,\infty)$	
a	0
ax	a
x^a	ax^{a-1}
b^x	$b^x \ln b$
e^x	e^x
$\log_b x, \ x \in (0,\infty)$	$\frac{1}{x \ \ln b}$
$\ln x, \ x \in (0,\infty)$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x, \ x \epsilon \mathbb{R} \backslash \left\{\frac{\pi}{2} + k\pi\right\}, \ k \epsilon \mathbb{Z}$	$\frac{1}{\cos^2 x}$
$\cot x, \ x \epsilon \mathbb{R} \backslash \{k\pi\}, \ k \epsilon \mathbb{Z}$	$-\frac{1}{\sin^2 x}$
$\arcsin x, \ x \epsilon (-1,1)$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x, \ x \epsilon (-1,1)$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{x^2+1}$
$\text{arccot}\, x$	$-\frac{1}{x^2+1}$

Wzory na całki

$f(x)$	$\int f(x)dx$
Niech: $a \epsilon \mathbb{R} \backslash \{-1\}$, $b \epsilon (0,1) \cup (1,\infty)$, $d > 0$, $C \epsilon \mathbb{R}$	
0	C
1	$x + C$
x^a	$\frac{1}{a+1} \cdot x^{a+1} + C$
$\frac{1}{x}$	$\ln \lvert x \rvert + C$
b^x	$\frac{1}{\ln b} \cdot b^x + C$
e^x	$e^x + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\frac{1}{\cos^2 x}, \ x \epsilon \mathbb{R} \backslash \left\{\frac{\pi}{2} + k\pi\right\}, \ k \epsilon \mathbb{Z}$	$\tan x + C$
$\frac{1}{\sin^2 x}, \ x \epsilon \mathbb{R} \backslash \{k\pi\}, \ k \epsilon \mathbb{Z}$	$-\cot x + C$
$\frac{1}{\sqrt{1-x^2}}, \ x \epsilon (-1,1)$	$\arcsin x + C = -\arccos x + C$
$\frac{1}{\sqrt{d^2-x^2}}, \ x \epsilon (-d,d)$	$\text{arc}\sin\left(\frac{x}{d}\right) + C$
$\frac{1}{x^2+1}$	$\arctan x + C = -\text{arccot}\, x + C$
$\frac{1}{x^2+d^2}$	$\frac{1}{d} \cdot \arctan\left(\frac{x}{d}\right) + C$

Ułamki proste

Funkcja: „ułamek prosty pierwszego rodzaju”	Funkcja: „ułamek prosty drugiego rodzaju”
$f(x) = \frac{A}{(x-a)^k}, \ \ a, A \epsilon \mathbb{R}, \ \ k \epsilon \mathbb{N} \backslash \{0\}$	$f(x) = \frac{Bx + C}{(x^2+bx+c)^k}, \ \ b, c, B, C \epsilon \mathbb{R}, \ \ b^2 - 4c < 0, \ \ k \epsilon \mathbb{N} \backslash \{0\}$