Ruprecht-Karls-Universität Heidelberg

Institute for Mathematics and
Interdisciplinary Centre for Scientific Computing (IWR)

Prof. Dr. Anna Marciniak-Czochra
Applied Analysis and Modelling in Biosciences

Im Neuenheimer Feld 205
D-69120 Heidelberg
Telephone: +49 (6221) 54-12120
E-mail: anna.marciniak@iwr.uni-heidelberg.de
Web page: http://www.biostruct.iwr.uni-hd.de

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Review of the Doctoral Dissertation

Title: *Discrete neuron models from the point of view of dynamical system theory*
Author: Frank Fernando Llovera Trujillo, Gdańsk University of Technology

Scope and aims of the dissertation

The dissertation examines map-based neuron models and analyses them using methods from dynamical systems theory. The author investigates the dynamics of discrete maps that describe neuronal excitability, focusing on phenomena such as chaos, periodic cycles, bifurcations, rotation structures, and Lorenz-type return maps. The motivation for this analysis lies in the potential applicability of such models which, despite their high level of abstraction, are capable of capturing key aspects of neuronal behaviour, including excitability, spike generation, and transitions between regular and chaotic activity.

The research concentrates on the topological and bifurcation properties of the considered systems, in particular on the Chialvo and Courbage–Nekorkin–Vdovin (CNV) models and, for the latter, their reduction to Lorenz maps. The aim is to deepen the understanding of chaotic and periodic behaviour in discrete neuron models and to develop new mathematical tools enabling their systematic analysis. Although the dissertation does not include direct biological data, the author makes an effort to link the theoretical results to biological context. Analysis is supported by numerical simulations, partially made possible by original algorithms developed by the candidate.

Results

The dissertation offers a comprehensive analysis of discrete neuron models, focusing primarily on the Chialvo model and the Courbage-Nekorkin-Vdovin (CNV) model, complemented by preliminary work on a hybrid threshold system.

For the Chialvo model, studied in both its one- and two-dimensional versions, the author establishes that the one-dimensional reduction is S-unimodal. This insight enables the use of the classical theory of unimodal maps and leads to a full analytical classification of chaotic and periodic behaviour. The dissertation further identifies flip and fold bifurcations and discusses how the bifurcation structure of the reduced model shapes the dynamics of the full two-dimensional system, including the emergence of bursting phenomena.

For the CNV model, explored in both its piecewise-linear and cubic nonlinear forms, the author proves the existence of invariant intervals and analyses regions of chaotic and periodic dynamics. By applying the theory of Lorenz-type maps, the dissertation provides a systematic classification of spiking and rotation patterns and shows how these patterns arise through the underlying return-map structure.

A further contribution is the development of new algorithms for computing periodic-orbit itineraries in Lorenz-like maps, together with open-source implementations. The dissertation also demonstrates that many neuron models contain embedded Lorenz-type subsystems, thereby opening new avenues for the theoretical study of neuronal dynamics.

In addition, the work includes preliminary numerical results for the MHR hybrid threshold model, focusing on spike counts and the nature of the attractors.

Overall, these results not only advance the mathematical understanding of discrete neuron models but also offer theoretical interpretations of spiking and bursting behaviours and explain how parameter changes influence spike frequency and stability. While such interpretations are theoretical in nature, they provide valuable insights for the broader development of mathematically motivated neuron modelling.

Structure and format of the dissertation

The dissertation consists of four published research articles, preceded by an introduction and a theoretical preliminaries section, and supplemented with a chapter containing preliminary results. The included articles are:

(1) Periodic and chaotic dynamics in a map-based neuron model, *Mathematical Methods in the Applied Sciences* 46(11), 2023, 11906–11931.

(2) Spike patterns and chaos in a map-based neuron model, *Applied Mathematics and Computer Science (AMCS)* 33(3), 2023, 395–408.

(3) Analysis of dynamics of a map-based neuron model via Lorenz map, *Chaos* 34(4), 2024, 043110.

(4) On computing periodic orbits itineraries for Lorenz-like maps, *Mathematica Applicanda* 52(2), 2024, 245–269.

Each article corresponds to one chapter and together they form a coherent body of work on the dynamics of discrete neuron models. The dissertation also contains an extensive survey section covering bifurcation theory, topological dynamics, ergodic theory, S-unimodal maps, and Lorenz maps. Three of the four publications are co-authored, while the last one is single-authored. All papers are mathematical in nature, with a strong emphasis on dynamical systems theory and discrete neuron models.

It is also worth noting that the first three papers have already been cited. The single-authored paper has had less time to gain visibility due to its recent publication. The journals are of moderate standing, but having four publications at the doctoral stage is a substantial achievement.

Summary and Conclusion

The dissertation is predominantly theoretical, focusing on the mathematical analysis of neuron models, with only limited biological interpretation. Nevertheless, the theoretical contribution is exceptionally strong: the author employs both analytical and numerical methods and obtains results that are novel and meaningful in several respects. These findings link rigorous mathematical theory with numerical observations and may form the basis for future work on phenomena relevant to real neuronal systems. In my view, the dissertation clearly belongs to the domain of biologically motivated mathematics and is both scientifically compelling and methodologically sound. A notable strength of the dissertation is the use of numerical methods not only for illustration but also for the development of computational algorithms for analysing periodic orbits, which adds breadth to the presented approach. The work is grounded in peer-reviewed publications, and the accompanying self-report is carefully and thoughtfully prepared.

For these reasons, I am fully convinced that Mr **Frank Fernando Llovera Trujillo merits the award of the doctoral degree** (stopień doktora nauk matematycznych), and I would also like to recommend his dissertation for distinction.

Sincerely,

Anna Marciniak-Czochra