Out of Plane Auxetic Behaviour as a Result of Perforations in Thin Structures

E. R. Grima, J. N. Grima, J. N. Grima

Faculty of Science, University of Malta
Msida MSD 2080, Malta

auxetic@um.edu.mt

Auxetics exhibit the unusual property of getting fatter when stretched and thinner when compressed. Over the years various models and mechanisms which result in negative Poisson's ratios have been proposed. This work aims to highlight the fact that this anomalous behaviour may in fact be obtained through a very simple mechanism which involves thin planar systems (e.g. a sheet of paper) containing slits as illustrated in Figure 1. When such systems are pulled, the deformations will be such that the thin structure will bend out of plane to a folded geometry, an effect which results in a significant opening up in the third direction, i.e. giant negative Poisson's ratios. This effect may be easily observed using paper models as illustrated in Figure 2.

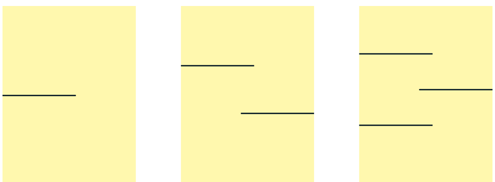

Figure 1: Examples of slits in thin sheets which can generate negative Poisson's ratios in the out of plane direction

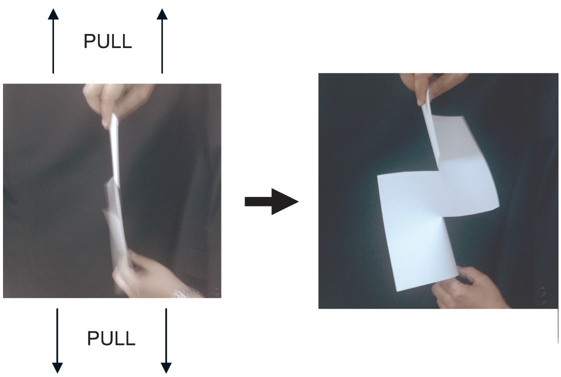

Figure 2: A 'proof of principle' that thin planar systems (in this case a sheet of paper) containing slits which can exhibit giant negative Poisson's ratios as they deform out of plane

Auxetic Foams and Models

J. N. Grima[1], B. Ellul[1], D. Attard[1], R. Gatt[1], E. Chetcuti[1],
E. Manicaro[1], M. Camilleri[1], T. Debono[2]

[1] *Faculty of Science, University of Malta*
Msida MSD 2080, Malta

[2] *Methode Electronics (Malta) Ltd.*

auxetic@um.edu.mt

Auxetic foams are special foams which exhibit the unusual property of expanding when stretched and getting thinner when compressed. This property results in several beneficial or enhanced properties such as an increased resistance to indentation, enhanced vibration absorption properties and a natural ability to adopt dome shaped surfaces. In this paper we discuss the available methods of manufacture of such foams and the conditions where they may be of use. We also discuss some of the models which are known to be able to explain the presence of auxeticity in such foams, in particular highly ordered models based on re-entrant cells and ones involving rotations of rigid joints. In view of the disordered microstructure which is characteristic of auxetic foams, we also discuss the effect of disorder on such models. We show that the existing highly ordered and symmetric models may have limitations in explaining the Poisson's ratio of real systems as the properties predicted by such models are significantly affected through the introduction of random defects.

Acknowledgements
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Modelling and Testing of a Foldable Macrostructure Exhibiting Auxetic Behaviour

J. N. Grima[1a], R. Galea[1], N. Ravirala[2], B. Ellul[1], D. Attard[1], R. Gatt[1], A. Alderson[2], J. Rasburn[3], K. E. Evans[3]

[1] *Faculty of Science, University of Malta*
Msida MSD 2080, Malta

[2] *Institute for Materials Research and Innovation, University of Bolton*
Deane Road, Bolton BL3 5AB, UK

[3] *College of Engineering, Mathematics and Physical Sciences*
Harrison Building, University of Exeter
Exeter, EX4 4QF, UK

[a] *joseph.grima@um.edu.mt*

Auxetics exhibit the anomalous property of getting fatter rather than thinner when uniaxially stretched (negative Poisson's ratio). This paper presents an analytical model and Finite Elements modeling of a structure constructed from umbrella type sub-units which is predicted to exhibit this property together with the results of mechanical tests of the structure shown in Figure 1 having this geometry. It is shown that such systems can exhibit negative Poisson's ratio the magnitude of which can be controlled by varying the geometric parameters associated with the structure. This means that these systems can be tailor-made to exhibit pre-desired properties so as to fit particular practical applications.

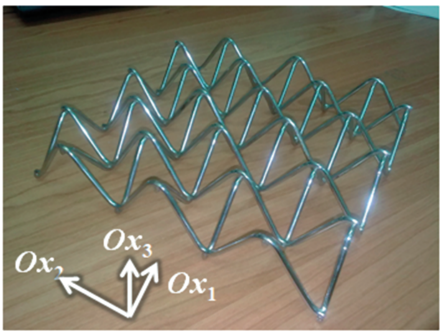

Figure 1: Geometry of the structure

Perforated Sheets Exhibiting Auxetic Behaviour

J. N. Grima, R. Gatt, B. Ellul, E. Chetcuti

Faculty of Science, University of Malta
Msida MSD 2080, Malta

Sheets made from readily available conventional materials containing star or diamond shaped perforations are simulated through FE models and are shown to be capable of exhibiting auxetic behaviour. This property may be exhibited in both tension and compression and can be explained through models based on rotating rigid units. We also show that through careful choice of the shape and density of the perforations, one may control the magnitude and sign of the Poisson's ratio. All this provides an easy and cost-effective method for the manufacture of systems at any scale which can be tailor made to exhibit particular values of the Poisson's ratio (auxetic or non auxetic) so as to fit particular practical applications.

Negative Pressure Dependence of Bulk Modulus of $Fe_{72}Pt_{28}$ Invar Alloy

G. Hausch

Dentalex
D 63505 Langenselbold, Germany

Usually the pressure derivative of the bulk modulus of solids is positive. Invar type alloys are known to exhibit unusual properties, e.g., the thermal expansion coefficient is reduced. For an $Fe_{72}Pt_{28}$ invar alloy which shows most pronounced invar anomalies, the thermal expansion coefficient is negative [1].

The bulk modulus of invar alloys also shows unusual behavior as a function of temperature a softening effect is observed which is the largest around the Curie temperature. Again, this softening effect is the largest for $Fe_{72}Pt_{28}$ [2]. It is shown that this softening effect is responsible in that the bulk modulus is lowered by pressure, i.e., this invar type alloy becomes easier to squeeze under pressure, i.e., the pressure derivative is negative [3, 4]. This behavior is observed in a temperature range around the Curie temperature.

Much work has been done to understand this effect theoretically. In thermodynamics we have to deal with two bulk moduli, defined as B at constant magnetic field (B_H) and B at constant magnetization (B_M). This is comparable to the specific heat where we deal with the specific heat at constant volume, C_v, and at constant pressure, C_p.

In theoretical treatments it is C_v that is relevant and not C_p which is experimentally observed.

Thus, correction terms, $C_p - C_v$, have been introduced.

The same problematic occurs in the case of the bulk modulus. The bulk modulus which is observed experimentally is B_H, but in thermodynamics the relevant modulus is B_M. Thus, a correction term, $B_H - B_M$, must be introduced [2] which is given by $1/B_H - 1/B_M = Vh \exp 2/X$, where h is the forced volume magnetostriction and X is the magnetic susceptibility.

In noninvar alloys this correction term can be neglected. However, in invar alloys this correction term is extremely large and when it is subtracted from the measured data the anomaly nearly disappears, i.e., the negative pressure dependence of the bulk modulus is nonintrinsic in nature. In other words, the negative pressure derivative of the bulk modulus of the $Fe_{72}Pt_{28}$ invar alloy is connected with the modulus B_H while the modulus B_M which describes the intrinsic behavior does not show this behavior, i.e., dB_H/dP is negative while dB_M/dP is positive. When discussing the nature of the invar effect it is only B_M and not B_H that is relevant. It has to be pointed out that, therefore, the invar effect, as far as the elastic constants are concerned, have been incorrectly analyzed [3, 4].

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Development of Auxetic Fabrics Using Knitting Technology

H. Hu[a], Z. Wang, S. Liu, Y. Liu

Institute of Textiles and clothing
The Hong Kong Polytechnic University
Hunghom, Hong Kong

[a] *tchuhong@polyu.edu.hk*

Knitting is a widely used fabric manufacturing technology. Compared with other fabric fabrication technologies, such as weaving and braiding, knitting is characterized by its higher process flexibility and greater fabric structure availability. Especially,

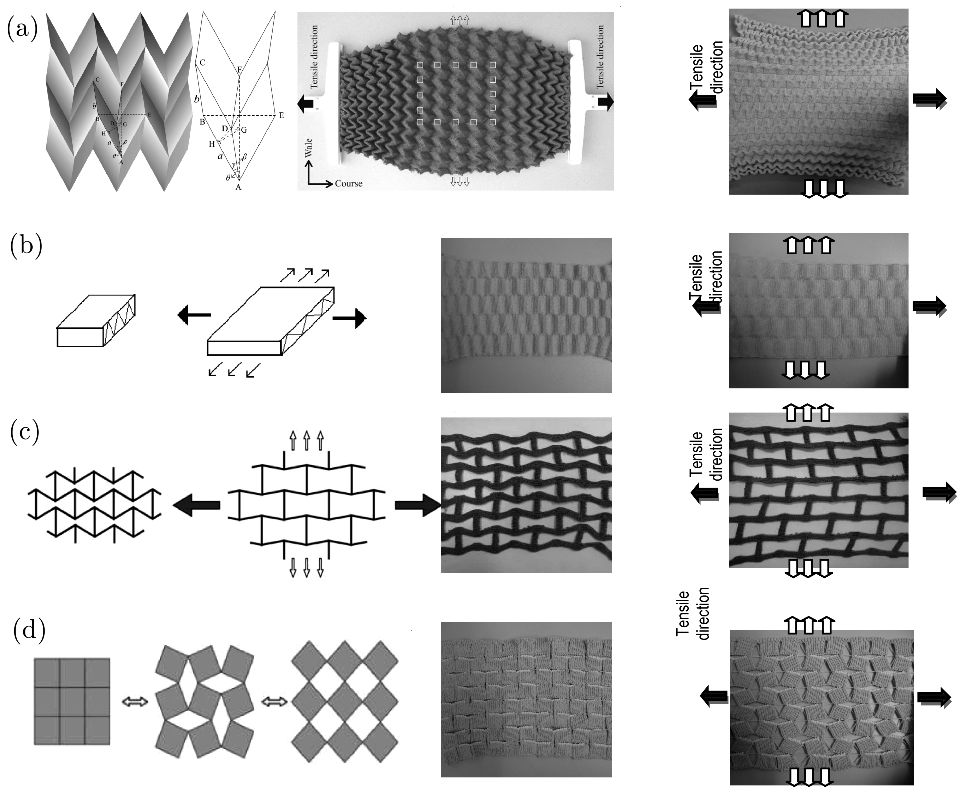

Figure 1: Knitted auxetic fabrics

knitting is very suitable to fabricate open mesh fabric structures as well as shaped two and three-dimensional fabrics. In this work, knitting technology has been explored to produce auxetic fabrics which laterally get big when stretched. Four different geometrical structures, which can realize the auxetic effect, were employed as basic reference structures for the development of knitted auxetic fabrics. These structures were a three-dimensional folded structure (Figure 1a), a three-dimensional spacer fabric structure (Figure 1b), a re-entrant hexagonal structure (Figure 1c) and a rotating structure with square units (Figure 1d). The special weft knitting processes were developed based on these structures and auxetic fabrics were produced using a computerized flat knitting machine. This work shows that auxetic fabrics can be realized through knitted structures and the knitting technology can provide a simple, but highly effective way of fabricating auxetic fabrics from conventional yarns.

Acknowledgements
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Propagation of Light Through a Fiber with a Negative Index Core

A. Joseph, K. Porsezian, R. V. J. Raja

Department of Physics, Pondicherry University
Pondicherry 605 014, India

We intend to carry out a NIM-fiber integration for fiber optics and identify the nonlinearity conditions prerequisite for soliton propagation through a fiber made of negative index core and positive index cladding. The pioneering demonstration of novel negative-index behavior with split ring resonators (SRRs) and wire structures was given in the microwave frequencies [1]. Armed with new nanofabrication techniques, researchers could artificially engineer sub wavelength structures and push up the operation frequency range of NIMs to THz and optical frequencies. At this critical juncture with the practically optical NIMs at our hands we have tried to visualize the role of NIMs in fiber optic communication. Considering the wave propagation in cylindrical coordinates and applying the separation of variable technique, we have obtained the characteristic equation as $J_1(q_1a)/q_1J_0(q_1a) = K_1(q_2a)/q_2K_0(q_2a)$ wherein the conditions of meta-fiber ($\mu_1 < 0$, $\epsilon_1 < 0$, $\mu_2 > 0$, $\epsilon_2 > 0$) are applied. Further, at our disposal we have optical metamaterial with negative index regime extending from 1500 to 1750 nm [2]. With the core chosen to be of negative refractive index as per Figure 1a and a cladding of positive refractive index with absolute magnitude less than that of core, we found the effective propagation constant using the eigen value eq. (1) utilizing scalar effective index method. From that effective β, one can end up in the determination of β_2 and n_{eff}. From the plot of β_2 obtained, one can see that effectively β_2 reaches the value of zero at 1.527 μm. For soliton propagation, balancing the dispersion and nonlinear lengths, considering a pulse propagating at wavelength 1.52 μm, β_2 takes the value -28.45 ps^2/m, the corresponding nonlinearity γ required

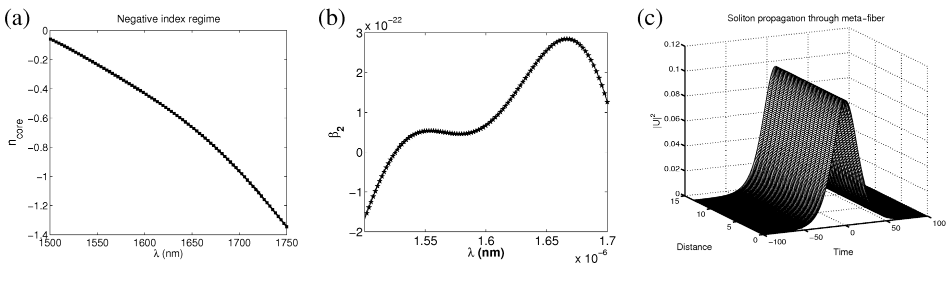

Figure 1: (a) Effective refractive index; (b) β_2; (c) soliton propagating through meta-fiber

for soliton propagation would be 0.7112 $m^{-1}W^{-1}$. Under this physical situation, the soliton propagation is shown in Figure 1c.

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Dispersion, Causality, and Negative Materials

P. Kinsler[a], A. Favaro, M. W. McCall

Blackett Laboratory, Imperial College London
Prince Consort Road, London SW7 2AZ, United Kingdom

[a] *p.kinsler@imperial.ac.uk*

Causality, dispersion, and loss are intimately connected by the Kramers-Kronig relations. Although this connection constrains what can be achieved by electromagnetic metamaterials, it also provides significant insight and enables useful calculations. We survey some recent results and describe how the principle of causality enshrined in the Kramers-Kronig relations can be put to practical use.

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Composite Materials with Negative-Stiffness Inclusions: Recent Advances in Theory and Experiment

D. M. Kochmann, T. Jaglinski, L. Dong, W. J. Drugan, R. S. Lakes

Department of Engineering Physics, University of Wisconsin-Madison
1500 Engineering Dr., Madison, 53706 Wisconsin, USA

The rigorous classical bounds of viscoelastic composite theory provide limits on the achievable composite properties (in particular, the overall stiffness and damping capacity) in terms of the properties and arrangements of the composite's constituents. These bounds result from the assumption, presumably made for stability reasons, that each constituent material must have positive-definite elastic moduli. If this assumption is relaxed, theoretical solutions have shown that these bounds can be greatly exceeded, resulting in advanced materials of extreme stiffness and damping capacity with enormous engineering potential.

In this paper, we address the two crucial aspects of stability and performance of such composites: On the one hand, we employ elastic stability theory to show that an elastic composite material with one phase having non-positive-definite elastic moduli (i.e., negative stiffness) can indeed be stable overall in the practically useful situation of applied traction boundary conditions. Hence, the classical stability criterion for elastic media can be relaxed in a composite. On the other hand, we give an overview of recent experimental findings that confirm and investigate the bound-exceeding viscoelastic performance of metal-ceramic composites (Sn-$BaTiO_3$), where the negative-stiffness effect is realized via phase-transforming particles trapped in a stiff metal matrix.

On the Mechanical Properties of Connected Rigid Rectangles: a Comprehensive Study

J. N. Grima, E. Manicaro, E. Chetcuti, D. Attard

Faculty of Science, University of Malta
Msida MSD 2080, Malta

Auxetics are materials that have a negative Poisson's ratio i.e. get fatter when uniaxially stretched and thinner when uniaxially compressed. This behaviour is the result of the manner in which particular geometric features in the materials' micro or nanostructure deform when they are subjected to uniaxial loads. Here, we discuss a model made from different sized rigid rectangles which rotate relative to each other. We show that such systems can exhibit scale-independent auxetic behaviour for stretching in particular directions, with the Poisson's ratios being dependent on the shape and relative size of the different rectangles in the model and the angle between them.

Complete Characterization and Synthesis of the Response Function of Elastodynamic Networks

F. Guevara Vasquez, G. W. Milton, D. Onofrei

Department of Mathematics, University of Utah
Salt Lake City, Utah 84112, USA

In order to characterize what exotic properties elastodynamic composite materials with high contrast consitituents can have in the continuum it makes sense to first understand what behaviors discrete networks of springs and masses can exhibit. The response function of a network of springs and masses, an elastodynamic network, is the matrix valued function $W(\omega)$, depending on the frequency ω, mapping the displacements of some accessible or terminal nodes to the net forces at the terminals. We give necessary and sufficient conditions for a given function $W(\omega)$ to be the response function of an elastodynamic network, assuming there is no damping. In particular we construct an elastodynamic network that can mimic any achievable response in the frequency or time domain. Our characterization is valid for networks in three dimensions and also for planar networks, which are networks where all the elements, displacements and forces are in a plane.

Anomalous Behavior of Constrained Auxetic Square in Two Dimensions

A. A. Poźniak[1a], B. T. Maruszewski[2b], T. Stręk[2c],
K. W. Wojciechowski[3d]

[1] *Institute of Physics, Poznan University of Technology*
Nieszawska 13a, 60-695 Poznań, Poland

[2] *Institute of Applied Mechanics, Poznan University of Technology*
Piotrowo 3, 60-695 Poznań, Poland

[3] *Institute of Molecular Physics, Polish Academy of Sciences*
M. Smoluchowskiego 17, 60-179 Poznań, Poland

[a] *aap@ifmpan.poznan.pl,* [b] *bogdan.maruszewski@put.poznan.pl*
[c] *tomasz.strek@put.poznan.pl,* [d] *kww@man.poznan.pl*

Rapid progress in new technology development requires new materials in order to realize new concepts. An important branch among variety of new materials are these of unusual mechanical properties expressed by means of elastic moduli – Young's modulus (E) and Poisson's ratio (PR, ν). Materials with negative PR, referred to as *auxetics*, are important and interesting both from engineering and scientific point of view [1–5]. Recently, a lot of effort have been put on revealing mechanisms responsible for auxeticity of materials.

Even the simplest model of elastic body – two-dimensional isotropic continuum – becomes an interesting object of investigation when subjected to some constraints. The aim of this work is to describe the behavior of the flat elastic square under such a boundary conditions that its left and right edges are fixed whereas the top and bottom edges are subjected to tensile (compressive) pressure. Simulations were conducted be means of Finite Element Method using both uniform and non-uniform meshes. The discretization was done with Lagrange elements of different orders. Obtained linear sparse system of algebraic equation was then solved be means of direct methods. Results were verified using both commercial and open-source software.

In the simulations, a counterintuitive effect was observed [6], analogous to that found previously in three dimentions [7]. The shape of the edges under tension (compression), unlike the case of common materials, i.e. those with positive PR, did not look like a function with one extremum – undergoing the tension (compression), but revealed at least *three* extrema – a minimum (maximum) and two maxima (minima). In other words, the displacement field locally (near the corners) had a component opposite to the acting force! It means that auxetics under such boundary conditions exhibit (locally) negative compressibility. Critical PR value $\nu_c(N)$, depending on mesh density (N) below which the edge was deformed in unusual way, was introduced. By iterative mesh refinement and extrapolation it was concluded that the mentioned strange effect does occur for *any* negative PR. Moreover, for values of PR close to its lower limit (-1) more than two "hills" were observed. It was shown that the lower value of PR, the denser meshes and the interpolating polynomial of higher orders are required to make the system convergent. Infinite number of "hills" in the case of PR $\rightarrow -1$ (for infinitely dense mesh) is expected.

Acknowledgements
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Spinning a New Yarn:
Fabricating Auxetic Yarns from Nonauxetic Fibers

A. M. Romine

Unionville High School
106 Blackshire Road, Kennett Square, 19348, Pennsylvania USA

This experimentation is novel research into a new class of materials, which may provide unique benefits in a wide-range of applications including superior tennis racquets. The following engineering goals were achieved:

1. Creating auxetic yarns through helical wrapping of two nonauxetic yarns;
2. Observing macrostructural changes indicative of auxetic behavior when placed under tensile load;
3. Quantitatively measuring key physical properties of auxetic yarns and nonauxetic fibers using an Instron® device; and
4. Devising a method and creating an apparatus that simulates a tennis racquet to measure the energy transfer and recovery properties of the test yarns when hit with a tennis ball

Two bicomponent types of auxetic fibrous yarns were created by wrapping high modulus Kevlar® around low modulus nylon and around low modulus Sorona®. Observations of strain confirmed the auxetic lateral expansion in the new yarns.

Quantitative measurements of the width of each yarn under strain from 0 g to 500 g showed the greatest auxetic change to Kevlar®/Sorona® wrap, expanding 124% in width.

Measurements of tenacity-at-break showed that neither auxetic yarn Kevlar®/Sorona® (12.34 gf/den) nor Kevlar®/nylon (12.78 gf/den) significantly compromised the strength of the Kevlar®(14.2 gf/den).

An apparatus was designed, created, redesigned, and refined, to measure the height of a tennis ball's bounce off of each fibrous yarn when woven like a tennis racquet. Kevlar®/Sorona® yarn performed better than any unwrapped yarn tested with bounce heights 105% compared to the highest bounces on unwrapped fiber, Kevlar®.

Auxetic fibrous yarns demonstrated potential enhancements for applications similar to tennis strings.

Developments in a Hip Stem Having Auxetic Properties

M. Sanami, K. L. Alderson, A. Alderson

University of Bolton, Institute for Materials Research and Innovation
Deane Road, Bolton BL3 5AB, UK

ms2cmr@bolton.ac.uk

Hip implants are used to relieve hip pain resulting from hip disease, cartilage wear and osteoporosis. There are two main types of fixation: cemented and porous [1–3]. The porous surfaces on the hip replacement implant are designed to engage the bone within the canal and permit bone to grow into the porous surface. Eventually, this bone in-growth can provide additional fixation to hold the implant in the desired position. Total hip replacement includes four main parts: acetabular shell, liner, femoral head and femoral stem. Dynamic stresses are generated due to human activity, therefore it is important to ensure the hip prostheses against static, dynamic and fatigue failure. The Finite Element (FE) method is an important tool in the design and analysis of total joint replacements [4, 5]. In this study, various two-dimensional auxetic and non-auxetic structures were selected and assessed for application as a basic hip prosthesis. Stress analyses of the structures were performed using the ANSYS FE code. Three-dimensional stem models were then created with homogenized properties determined from the auxetic and non-auxetic structures, and static analyses were conducted under body load for the stems.

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Gradient Auxetic Cellular Structures

F. Scarpa, S. Hummel

*Department of Aerospace Engineering, University of Bristol
BS8 1TR Bristol, UK*

f.scarpa@bris.ac.uk

We describe the design, fabrication modeling and testing of auxetic cellular structures with gradient unit cell layouts. Gradient auxetics allow a continuous and differentiate sinclastic curvature, with the possibility of manufacturing sandwich structures with complex shapes other than saddle and dome-like ones [1]. We have designed and

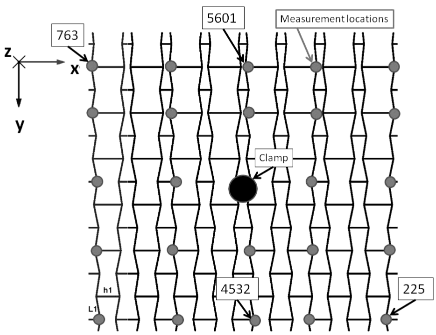

Figure 1: Gradient auxetic cellular panel

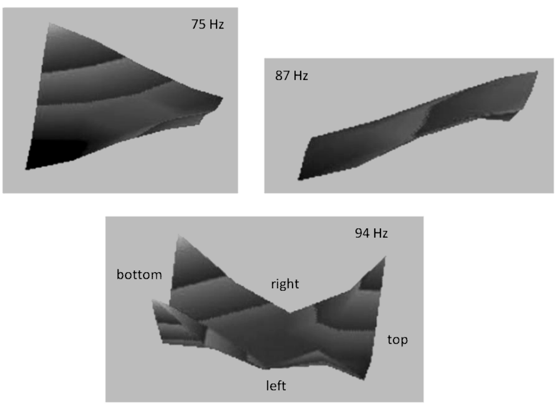

Figure 2: Operational mode shapes of the gradient panels

produced angle and aspect ratio gradient topologies, manufactured using RP FDM techniques. The dynamics properties (modal analysis and frequency response functions) have been simulated using full FEM and a Component Mode Synthesis (CMS) approach, to reduce the order of the numerical models and increase the computational efficiency. The samples have been subjected to forced vibration tests with data acquired with scanning laser vibrometry. We observe good correlation between the numerical and experimental results, and provide comments regarding the influence of the geometry layouts over the modal densities of the gradient honeycomb panels.

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Resonant Multimeander-Metasurfaces: Model System for Superlenses and Communication Devices

H. Schweizer[1], L. Fu[1], T. Weiss[1],
P. Schau[2], K. Frenner[2], W. Osten[2], H. Giessen[1]

[1] *4th Physical Institute, Universität Stuttgart*
Pfaffenwaldring 57, 70550 Stuttgart, Germany

[2] *Institute für Technische Optik, Universität Stuttgart*
Pfaffenwaldring 9, 70569 Stuttgart, Germany

Resonantly coupled metasurface structures can solve the problem of optical losses present in negative index materials used for subwavelength imaging and for functional structures in the communication technology. In this contribution a multi metasurface structure consisting of single *meander meta surfaces* (MMS) is investigated. The structure can serve for subwavelength imaging in microscopic applications as well as for optical wave guiding in communication technology applications.

With MMS structures we follow a novel metasurface concept by introducing three dimensional cell elements via the meander structure in the metasurface plane. This gives an independent degree of freedom for the design of the metasurface frequency spectrum. We can show that the resonance frequencies of the electrical and magnetical dipole moments of the MMS structure can be designed by relatively independent geometrical degrees of freedom. Furthermore the MMS structure allows for an efficient coupling of the electric and magnetic fields of a light wave at perpendicular and oblique incidence.

An analysis of single uncoupled MMS structures by a rigorous coupled-wave approach (RCWA) reveals that the plasmon dispersion of single MMS structures contains contributions of long range surface plasmon polaritons (LRSPP) and short range surface plasmon polaritons (SRSPP) which directly contribute to the magnetic and electric dipole moments in the optical response. The structure analysis according to the transmission line method indicates that the SRSPP belongs to the magnetic dipole and the LRSPP to the electric dipole moment in a meander structure. The interplay between these dipole moments results in a resonant transmission passband and the possibility to obtain interesting material parameters such as epsilon near zero, negative index or resonances in the dielectric constant useful to reduce the velocity of light for optical delay line purposes.

An analysis of a resonantly coupled multi MMS structure reveals a mode system of surface plasmon cavity polaritons where the local SPP mode of the uncoupled MMS dominates the cavity polariton in the multiple MMS structure. Thus, the MMS behaves as a frequency selective cavity mirror in the resonantly coupled metasurface structure. Furthermore, a resonant energy transfer between metasurfaces in multiple structures can be observed for nearly unlimited numbers of metasurface stacks. This is a key property for obtaining far field superlenses.

Far field super lenses transform the subwavelength information coded in evanescent waves of the object into travelling waves to form the far field image. This could be obtained in a resonantly coupled MMS structure. First, the evanescent waves of the object excite plasmonic excitations in the first MMS which are scattered into travelling waves of a higher diffraction order by the periodic structure. Then, they are transferred through the MMS and coupled out to the far field enabling the obtaining of a far field superlens. It is also novel structures for communication devices such as polarization beam splitters and delay lines from double MMS that can be obtained by placing the metastructure in a conventional waveguide. A possible implementation will be discussed.

Mechanical Characterisation of Auxetic Cellular Metals Produced by Selective Electron Beam Melting (SEBM)

J. Schwerdtfeger, F. Schury, M. Stingl, R. F. Singer, C. Körner

Lehrstuhl Werkstoffkunde und Technologie der Metalle, Universität Erlangen-Nürnberg Martensstraße 5, D-91058 Erlangen, Germany

Research activity in the area of auxetic structures has intensified considerably during recent years as they are seen as candidates for a wide range of applications. Their auxetic nature is always the result of complex internal geometries which are very difficult to produce through traditional manufacturing routes. In our work we have chosen an additive manufacturing approach to produce auxetic cellular metal structures from pure titanium and titanium alloy (Ti-6Al-4V) [1]. The selective electron beam melting system (SEBM) is a powder bed based system in which structures are build up layer wise using an electron beam to fully melt the material in the desired places. The geometric data for every layer is extracted from a 3D-CAD file of the structure. This approach allows us to build fully dense parts of arbitrary geometry from metal powders, giving a tremendous degree of freedom in design of auxetic cellular structures. The high degree of control over the mesostructure also results in a high degree of control over the mechanical properties of the cellular material. In our paper we will present experimental and numerical results on the mechanical properties of a self designed 3D auxetic structure (see Figure 1) [2]. The structures were assessed using CT-scans and their mechanical properties were determined experimentally through

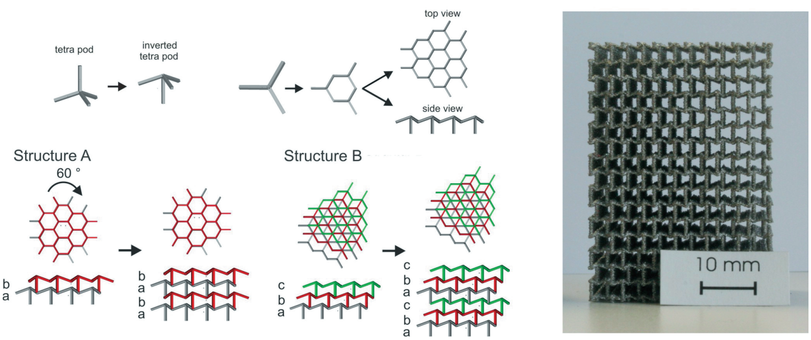

Figure 1: New 3D auxetic structure based on inverted tetrapods. Schematic build plan of the structure (left) and the finished structure produced by SEBM (right) are shown

compression testing. The auxetic nature of the structure could be confirmed. Numerically the structure was modelled using finite element simulations and the influence of orientation and relative density on the mechanical properties including its negative Poisson's ratio was thoroughly investigated. We will also present first results on auxetic cellular structures identified through topological optimisation.

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Joint Cartilage as an Auxetic Material

S. V. Shilko[a], E. M. Petrokovets, Y. M. Pleskachevsky

V. A. Belyi Metal-Polymer Research Institute of National Academy of NASB
32A Kirov Str., 246050 Gomel, Belarus

[a] *Shilko_mpri@mail.ru*

Biological tissues, such as joint cartilage, are extremely efficient structures with respect to optimization of functional characteristics, mainly, frictional and mechanical ones [1]. Such factors should ideally be reproduced in endoprostheses implanted during surgical treatment of human joint diseases. In particular, the explanation of remarkable properties of cartilage consisting of porous matrix structures filled with a liquid crystal substance (sinovia) has been presented in [2] in terms of an adaptive bi-phase structure with moving interface. The auxetic phenomena in this biotissue seem quite preferable for in terms of the properties mentioned above and may be taking into account in creating adequate implants for replacing damaged organs [3]. This hypotheses investigated in this paper by means of computer simulations involves the deformation of the cartilage structure as a bi-phase composite having negative Poisson's ratio ν.

In order to predict the deformation behavior of cartilage under contact loading, finite element models have been developed. During the preliminary numerical tests, the contact deformation and stress-strain state of elastic collagen matrix filled by quasi-liquid low modulus phase (sinovia) have been examined. For the simulation of open-cell structure of matrix we use a large number of cubic cells. Symmetry of the joint model allows one to represent the axial-symmetrical statement of the contact problem.

The dependence of contact pressures, tangential stresses and displacements as well as internal stresses on the Poisson's ratio ν of each phase has been identified. The effect of compression-driven synergic self-lubrication and self-hardening of the model auxetic cartilage structure has been predicted. Finally, the possibility of artificial cartilage made of auxetic material is discussed.

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Magnetotunable Plasmon-Polaritons in Negative Index Semiconductor Superlattices

R. H. Tarkhanyan

Institute of Radiophysics and Electronics
Ashtarack-2, 378410, Armenia

rolandtarkhanyan@yahoo.com

A detailed study of long-wavelength magnetic plasmon-polaritons in nanoscale semiconductor superlattices is presented. The necessary conditions for the negative refraction (NR) are obtained in the presence of an external magnetic field. Classical semiconductor superlattices (SL) are periodic multi-layer nanostructures constructed by alternating planar layers of two or more different materials at least one of which is a semiconductor. For the first time, SL have been proposed by Esaki and Tsu [1] as artificial crystals with a periodicity much larger than the lattice constants of the constituent components. In fact, SL are a specific class of metamaterials – artificial nanocomposites with unusual electromagnetic properties not observed in nature, such as negative refraction, superlensing phenomenon, inverse Cherenkov radiation, reversed Doppler shift, etc. Although the concept of NR has existed for more than six decades, the actual incitement for all the recent activities in the optics and technology of metamaterials which exhibit NR and near field focusing has been set by Veselago [2] and Pendry [3]. In this presentation, three different types of SL are considered including those consisting of alternating layers of nonmagnetic semiconductors and magnetic insulators (ferrites and/or antiferromagnets). Interaction of electromagnetic waves with magnetoplasmons in semiconductor layers and with spin-lattice vibrations in magnetic layers are taken into account. When characteristic frequencies of these vibrational subsystems are comparable, the mutual interaction becomes especially strong and then collective modes of coupled magnon-plasmon polaritons appear, the dispersion properties of which are very sensitive to the magnetic field application. The effective permittivity and permeability tensors are derived for the magnetic field's perpendicular and parallel orientation with respect to the plane of the layers and the influence of the magnetic field on the wave refraction in ferrite-semiconductor superlattices is examined. The frequency bands supporting anomalous refraction of the electromagnetic waves are identified. Both the phase and group refractive indices are determined analytically and the time averaged Poynting vector is calculated. It is shown that in the structures containing antiferromagnetic layers, the number of the polariton branches in electromagnetic spectrum, as well as dispersion behavior of the magnonic excitations (i.e., coupled magnon-plasmon polaritons) in different ranges of the static magnetic field strength are quite diverse.

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Negative Poisson's Ratio in a Two-Dimensional Lennard-Jones System

K. V. Tretiakov[1a], K. W. Wojciechowski[2b]

[1] *Institute of Molecular Physics, Polish Academy of Sciences*
M. Smoluchowskiego 17, 60-179 Poznań, Poland

[2] *PWSZ im. Prezydenta St. Wojciechowskiego*
Nowy Świat 4, 62-800 Kalisz

[a] *kvt@ifmpan.poznan.pl*, [b] *kww@man.poznan.pl*

Common materials show positive Poisson's ratio, i.e. shrink transversally when stretched. Materials showing opposite behaviour, i.e. expanding transversally at the stretch, have been first manufactured more than two decades ago [1] and are the subject of increasing interest [2]. Studies of simple models allow one for better understanding of those unusual materials.

Two-dimensional Lennard-Jones system was investigated by constant pressure Monte Carlo simulations with variable shape of the periodic box [3]. Elastic moduli and Poisson's ratio of the system were determined by analyzing box fluctuations. In agreement with theoretical predictions [4, 5], it was found that in a range of negative pressures (uniform tension) the system exhibits *auxetic* behaviour [6], i.e. shows negative Poisson's ratio.

Acknowledgements

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Method of Fundamental Solutions for Solving Wave Propagation Problem in Auxetic Material Plate

A. Uściłowska

Institute of Applied Mechanics, Poznan University of Technology
Piotrowo 3, 60-965 Poznan

anita.uscilowska@put.poznan.pl

This paper describes the application of the method of fundamental solutions to solve the wave propagation in a plate made of auxetic material. It is assumed that the wave is a localized bell-shaped strain of a permanent form (solitary wave). The problem is an initial-boundary value problem and it is presented by a partial differential equation of fourth order [1]. The equation has a nonlinear form, therefore, combined techniques are implemented to solve the initial-boundary value problem.

In the first step of the proposed numerical approach it is assumed that the time domain is descretized and the solution is calculated at some chosen time steps. The proposal of the paper is to approximate the partial differential with respect to the time variable by finite difference. In this way a boundary-value problem is to be solved at every time step. The problem is described by a nonlinear partial differential equation and linear boundary conditions. Picard iterations are implemented to solve the problem of the nonlinearity of the equation. Therefore, an iterative algorithm is applied for each time step.

Moreover, at every iteration step, a linear boundary problem is obtained and it is to be solved. For this purpose the Method of Fundamental Solution (MFS) is implemented. The MFS (see [2, 3]) requires knowing a special function related to the operator appearing in the equation which is called the fundamental solution. The considered equation is the fourth order partial differential, however, the fundamental solution thereof is unknown. Therefore, a special technique is proposed to expand the equation to include a biharmonic operator. Implementation of the Method of Fundamental Solution to solve a problem with an equation with a biharmonic operator and a boundary condition is known in the literature [4].

A numerical experiment is performed to confirm the convergence and accuracy of the proposed numerical algorithm.

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"Negative"-Diamagnetism of Three Dimensional Arrays of Semiconductor Nano-Rings

L. M. Thu, W. T. Chiu, O. Voskoboynikov

Department of Electronics Engineering
National Chiao Tung University
1001 Ta Hsueh Rd., Hsinchu 30010, Taiwan

Recent advances in the fabrication of semiconductor structures make it possible to design and study in detail the magnetic properties of different kinds of semiconductor nano-objects such as quantum dots and nano-rings. Under certain conditions those conventionally diamagnetic nano-objects changing the sign of magnetic susceptibility demonstrate a "negative"-diamagnetic response [1–3]. In this theoretical study we consider diamagnetism in three dimensional arrays of InAs/GaAs asymmetrical nano-rings (ANR) [1] (see inset in Figure 1). The electronic magnetic response of the rings can be found based on the effective one-electronic band Hamiltonian (energy-position-dependent electron effective mass approximation). In our simulations mapping the actual strain and material content in a nano-ring we use three-dimensional smooth confinement potentials for electrons [3]. The external magnetic field **B** is applied in the system's growth directions. At low temperature the differential magnetic susceptibility of an individual nano-ring has a positive peak (which is auxetic to the conventional diamagnetism). Once the magnetic response of an individual nano-ring is achieved, we estimate, using the Claussius-Mossotti equation [4], the effective susceptibility of three dimensional arrays composed of the rings. In Figure 1 we show

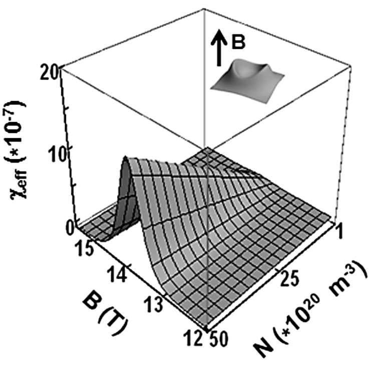

Figure 1: Effective susceptibility of arrays of nano rings

the effective susceptibility (χ_{eff}) of arrays with different volume concentration (N). The "negative"-diamagnetic (positive susceptibility) peak remains Lorenz-like shaped and gradually disappears when the ring's concentration decreases. This result demonstrates an opportunity to design semiconductor nano-structured materials with positive ("negative"-diamagnetic) effective susceptibility.

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Stability of Viscoelastic Continuum with Negative-Stiffness Inclusions in Low Frequency Range

Y.-C. Wang, C.-C. Ko

Department of Civil Engineering
National Cheng Kung University
Tainan, Taiwan 70101

Experimental findings on the negative-stiffness composites have shown unbounded overall viscoelastic stiffness and damping. Theoretical studies on composite materials under purely elastic assumptions have revealed that a negative-stiffness state in the inclusion is attainable, providing the surrounding matrix is sufficiently stiff. In the present work, viscoelastic composites with the standard-linear-solid constitutive relationship were analyzed for their unusual effective mechanical properties and stability. Two composites were studied. One was a three-dimensional composite consisting of a sphere in a cube, and the other was a two-dimensional system composed of a circular inclusion in a square plate. Both the composites were under uniaxial loading. For the 2D case, the plane strain assumption was applied. The finite element method was adopted to solve the viscoelasticity problems. The shear modulus of the inclusions was assumed to be positive to avoid banding instability, but other moduli were allowed to be negative. Effective modulus and damping were studied in correlation with the stability of the mechanical system. In the low frequency limit, the sufficient stability condition, based on the energy argument, derived from the elasticity theory may be used for validation. It was found that the extreme properties of 3D viscoelastic composites required negative stiffness in the unstable regime. However, the plane-strain 2D composite showed anomalous mechanical properties in the stable regime. The rationale may be due to additional displacment constraints from the out-of-plane direction. The viscoelastic stability conditions for the 2D and 3D composites were derived to be compared with purely elastic results.

On Some Mechanisms Leading to Negative Poisson's Ratio

K. W. Wojciechowski

Institute of Molecular Physics, Polish Academy of Sciences
M. Smoluchowskiego 17, 60-179 Poznań, Poland

kww@ifmpan.poznan.pl, kww@man.poznan.pl

Isotropic materials exhibiting negative Poisson's ratio, called *auxetics*, behave in a counterintuitive way. When stretched / compressed they not only increase / decrease their dimension along the direction of applied force but they do *the same* with their transverse dimensions. This anomalous feature of auxetics, as well as applications following it, make such materials a subject of increasing interest for researchers representing various fields of science and engineering. Finding out qualitatively different mechanisms leading to negative Poisson's ratio and understanding them is important not only from the point of view of science but also for technological reasons – it can result in discovering or manufacturing of new auxetic materials. Studies of models constitute a simple and efficient way to reach this goal.

In this lecture, some models of auxetics and mechanisms leading to auxetic behaviour will be presented and discussed. Anisotropic systems exhibiting negative Poisson's ratio in some directions only, called *partial auxetics*, will be also considered.

Modelling of LCP-Type Systems Exhibiting Negative Poisson's Ratios

C. Zerafa, J. N. Grima

Department of Chemistry, Faculty of Science, University of Malta
Msida MSD 2080, Malta

auxetic@um.edu.mt

Materials having negative Poisson's ratio (auxetic) exhibit the unusual property of expanding when uniaxially stretched and getting thinner when compressed, thus contrasting with conventional materials which show the opposite behaviour.

A. C. Griffin et al. has proposed and synthesised liquid crystalline polymers (LCPs) which have been reported to exhibit this behaviour. The auxeticity in these polymers arises as a result of a mechanism which involves re-orientation of laterally attached rod-like units which in the unstressed state are aligned in the direction of the main polymer chain, whilst in the stressed state they rotate to the orthogonal direction. This deformation mechanism results in a lateral expansion due to an increase in the inter-chain distance when the polymer is stretched.

This paper reports the results of molecular mechanics and molecular dynamics simulations performed using the PCFF force-field, on polymeric systems based on Griffin's LCPs in order to understand better the behaviour of these systems when they are stretched. These simulations suggest that when such polymers are stretched along the direction of the main chain (the Z-direction), the systems are conventional at small strains but auxetic in some planes at higher strains.

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